Incompletable Grounding and Ontological Economy Kelly Trogdon Sept. 20, 2024 Forthcoming in Analysis

1. Introduction

Roughly speaking, *incompletable* grounds are partial grounds that do not, together with other partial grounds, fully ground.¹ I first bolster the overall case for incompletable grounding by arguing that a certain totality fact has incompletable grounds. Then I trace out some interesting consequences for the ontological economy of theories, including those according to which the totality fact obtains.

2. Totality facts

Is there incompletable grounding? Some, including Elgin (2018), Leuenberger (2020), Trogdon and Witmer (2021), and Giannotti (2022), propose potential examples. Others note theoretical reasons for taking the idea seriously. For example, Emery (2021) points out that cases of indeterministic causation are naturally interpreted as involving contributory causes that do not, together with other contributory causes, comprise sufficient causes. So, if causation and grounding are broadly analogous (where contributing causes correspond to partial grounds and sufficient causes to full grounds), we have defeasible evidence that incompletable grounding is possible if not actual. In this section I bolster the case for incompletable grounding, arguing that a specific totality fact has incompletable grounds. And I contrast my proposal with two similar but less satisfactory proposals.

Suppose that $a_1, ..., a_n$ are in fact all the existing individuals. Consider the following "totality" fact:

(1) $[(\forall x)(x = a_1 \lor x = a_2 \lor ... x = a_n)].$

(1) is a totality fact in that it says that there are no more individuals than a_1 , a_2 , ... a_n . (1) is theoretically important, as we need to appeal to it in providing full grounds for various facts, including negative existential facts.

It seems that (1) is partially grounded. A fact is *fundamental* just in case it obtains and isn't partially grounded (Rosen 2010; Bennett 2011), and we have reason to think that (1) isn't fundamental. Some facts quantify over all individuals (e.g., $[(\forall x)(x = x)])$. (1) not only quantifies over all individuals but concerns every individual, as the sentence that expresses this fact contains a name for each individual. Microphysics (perhaps along with pure mathematics)

¹ The notion of incompletable grounding traces back to Fine's (2012) discussion of *strict partial* grounding.

serves as a guide to which sorts of facts are candidates for being fundamental. Since (1) concerns many individuals (e.g., ordinary macroscopic objects like tables) that fall outside the subject matter of microphysics (and pure mathematics), we have a (defeasible) reason to think that (1) isn't fundamental.

Okay, what facts partially ground (1)? Plausibly, any universal fact that isn't partially grounded by facts concerning essences or laws of nature is partially grounded by its instances. So, consider:

(2) $[a_1 = a_1 \lor a_1 = a_2 \lor \dots a_1 = a_n], [a_2 = a_1 \lor a_2 = a_2 \lor \dots a_2 = a_n], \dots [a_n = a_1 \lor a_n = a_2 \lor \dots a_n = a_n].$

Given that (2) specifies the instances of (1) and (1) isn't partially grounded by facts concerning essences or laws of nature, (2) partially grounds (1). And consider the following:

(3) $[a_1 = a_1], [a_2 = a_2], \dots [a_n = a_n].$

If $[\varphi]$ obtains, then $[\varphi]$ fully grounds $[\varphi \lor \psi]$. Hence, the facts among (3) fully ground the facts among (2). So, given the transitivity of grounding, (3) partially grounds (1).

At the same time, however, it seems that neither (2) nor (3) partially grounds (1) given the following widely endorsed general principles about grounding:

Completability: if Δ partially grounds $[\varphi]$, then there is some Γ such that Δ and Γ together fully ground $[\varphi]$.

Extractability: if Δ fully grounds [φ], then any sub-collection of Δ partially grounds [φ].

Necessitation: if Δ fully grounds $[\varphi]$, then it's necessary that if the facts among Δ obtain then $[\varphi]$ obtains.

Suppose, for reductio, that (3) partially grounds (1). Given completability, there is some Δ that together with (3) fully grounds (1). Given extractability, Δ is a partial ground for (1). And, given necessitation, (3) and Δ together necessitate (1). But, as I argue below, there is no Δ that together with (3) necessitates (1) while also partially grounding (1). NB: the argument could equally well be cast in terms of (2) rather than (3).²

As for completability, this principle follows from a widely endorsed definition of partial grounding in terms of full grounding: Δ partially grounds [φ] just in case Δ either on its own or

² In response, you might concede that (3) doesn't partially ground (1) but maintain that (1) is partially grounded, as this fact is *zero-grounded* (Muñoz 2022). If you (like me) are somewhat skeptical of zero-grounding and inclined to say that (2) and (3) partially ground (1), this response has limited appeal. See Amijee (2021) for critical discussion of the claim that (1) is zero-grounded.

together with other facts fully grounds [φ] (Fine 2012; Rosen 2010). As for the definition itself, proponents normally introduce it without explicit argument.

Turning to extractability, this principle also follows from the standard definition of partial grounding in terms of full grounding. And it enjoys independent support. Suppose, for reductio, that it's possible for full grounds to contain sub-collections that aren't partial grounds. In this case, it's possible for Δ to fully ground [φ], yet there is some sub-collection of Δ that doesn't partially determine/explain [φ]. But, if Δ fully grounds [φ], then Δ and any sub-collection of Δ partially determines/explains [φ] (Dasgupta 2014). (The same principle applies to merely partial grounds as well.)

Let's turn to necessitation. This principle is a relatively orthodox assumption, imposing a useful constraint on theorizing about grounding. As I see it, there are two main reasons you might reject it. First, you might, following Emery (2019) and Bader (2021), claim that some instances of full grounding are *stochastic* in nature, where such grounds don't necessitate what they ground. Second, you might, following Cohen (2020) and others, distinguish between grounds and *enablers* for grounds, where an enabler for a ground of some fact is understood to not itself be a ground of that fact. In this case we should maintain only that if Δ fully grounds [φ], then Δ and its enablers together necessitate [φ].

Suppose for the moment that necessitation is to be rejected for one of these two reasons. In this case, you might think that we can reasonably maintain that, while (3) doesn't necessitate (1), (3) fully (and thus partially) grounds (1). But (3) isn't a stochastic ground of (1) (at least given what Emery and Bader say about stochastic grounding). And there apparently are no enablers for (3) such that (3) and those enablers together necessitate (1), as I will argue below.

Let's now return to the claim that there is no Δ that together with (3) necessitates (1) while also partially grounding (1). Δ satisfies the *modal condition* just in case (3) and Δ together necessitate (1). So, the claim is that any Δ that satisfies the modal condition doesn't partially ground (1). Now, if Δ satisfies the modal condition, then Δ has one of two specific modal profiles:

- ∆ necessitates (1), or
- While Δ doesn't necessitate (1), (3) and Δ together necessitate (1).

I take it that (3) itself doesn't satisfy the modal condition, as there are various *horizontal* expansions of w—possible worlds in which a_1 , a_2 , ... a_n exist as well as further individuals (Bricker 2006).³ I'll consider some examples of facts with the modal profiles described above. Provided that these cases are representative, we have reason to think that no facts that satisfy the modal condition partially ground (1).

³ If (1) is necessary as Williamson (2013) maintains, is it plausible to maintain that (3) fully grounds (1)? Not in the absence of further argument—see Sider (2020: 39) for relevant discussion.

First modal profile: Δ necessitates (1). Perhaps the most obvious example of a fact with this modal profile is (1) itself. But (1) doesn't partially ground itself, given the irreflexivity of grounding. Consider instead:

(4) $[\neg(\exists x)\neg(x = a_1 \lor x = a_2 \lor ... x = a_n)].$

(4) necessitates (1) as they're logically equivalent. Does (4) partially ground (1)? It would seem not—plausibly, (1) and (4) are the same fact.

You might claim, following Armstrong (2004, 72), that there is a special "totaling" relation T that relates individuals and properties such that the following fact necessitates (1):

(5) $[T((a_1, a_2, ..., a_n), being an individual)].$

The idea is that $a_1, a_2, ..., a_n$ "total" the status of being an individual. As noted above, if Δ partially grounds $[\varphi]$, then Δ and any sub-collection of Δ partially determines/explains $[\varphi]$. But there apparently is no plausible story to tell (one appealing to either plausible general grounding principles or mediating mechanisms) according to which (5) plays such a role with respect to (1). And the same goes for the collection consisting of (3) and (5). Calling *T* the "totaling" relation doesn't help—this name reminds us of the theoretical role that (5) is supposed to play, but it doesn't shed any light on how (5) might do so. This calls into question not only the idea that (5) partially grounds (1), but whether this fact obtains in the first place.⁴

Second modal profile: while Δ doesn't necessitate (1), (3) and Δ together necessitate (1). No facts have this modal profile if the (3)-facts are all necessary. Suppose, then, that some identity facts concerning individuals are contingent. In this case, I take it that (3) is modally equivalent to the following:

(6) [*a*₁ exists], [*a*₂ exists], ... [*a*_n exists].

For any necessary existence fact among (6), it's corresponding identity fact among (3) is also necessary; and, for any contingent existence fact among (6), it's corresponding identity fact among (3) is also contingent. Now consider the following:

(7) $[\sim (a_1 = a_1 \& a_2 = a_2 \& ... a_n = a_n) \lor (\forall x)(x = a_1 \lor x = a_2 \lor ... x = a_n)],$

(8) [There are only *n* individuals].

Given that there are contingent facts among (3), (7) doesn't necessitate (1) on its own. But obviously (7) and (3) together necessitate (1). And the same considerations apply to (8).

⁴ Similar considerations apply to Fine's (2012) suggestion that there is a special non-quantificational fact that necessitates (1) and together with (3) fully grounds (1).

It's clear that (7) doesn't partially ground (1). As we've already noted, if $[\varphi]$ obtains, then $[\varphi]$ fully grounds $[\varphi \lor \psi]$. So, (1) partially grounds (7). It follows that (7) doesn't partially ground (1), given the asymmetry of grounding. On the face of it, (1) and (6) together fully ground (8). If this is right, then (1) partially grounds (8), given extractability. In this case, (8) doesn't partially ground (1), given the asymmetry of grounding.

Returning to the matter of enabling, reflection on the examples above also suggests that there is no enabler for (3), Δ , such that (3) and Δ together necessitate (1). Following Amijee (2021), (4) doesn't enable (3), provided that nothing enables something to ground itself. It would seem that (5) doesn't enable (3), as there apparently is no plausible story to tell about how it would do so. And neither (7) nor (8) enables (3), provided that Δ enables Γ to fully ground [φ] only if [φ] isn't a partial ground of facts among Δ .

Let's take stock of the discussion so far. I first argued that (3) partially grounds (1). But then I considered an argument that this isn't the case. How should we resolve this matter? Δ is an *incompletable* ground for $[\varphi]$ just in case Δ merely partially grounds $[\varphi]$, yet there is no collection of facts that includes Δ that fully grounds $[\varphi]$. I suggest that (3) is an incompletable ground for (1). And the same goes for (2). Returning to the argument above that neither (2) nor (3) partially grounds (1), the idea is to reject completability, the least well motivated and most theoretically dispensable of the general principles at issue in the argument.

You might worry that, even if (3) is an incompletable ground for (1), we face a similar puzzle concerning the grounding profile of facts among (3). Recall that one of our reasons for thinking that (1) isn't fundamental (and is therefore partially grounded) is this: among the individuals this fact concerns, many fall outside the subject matter of microphysics. As Shumener (2020) points out, the same considerations suggest that various facts among (3) such as [Beijing = Beijing] are non-fundamental as well. But, at the same time, it's hard to see just how such identity facts might be grounded.

While I don't have the space to fully address this issue, here's a view that strikes me as promising. Identity facts concerning individuals (whether they're part of the subject matter of microphysics or not) are fully grounded by existence facts concerning those individuals (where existence is treated as property, so the existence facts aren't quantificational in nature).⁵ But what about the grounding profile of the (6)-facts so understood? I take it that, for any existence fact that concerns an individual that isn't part of the subject matter of microphysics, that fact is at least partially grounded by facts that don't concern such individuals. (Such facts might concern individuals included in the subject matter of microphysics, or they might be qualitative rather than individualistic in nature.) One possibility is that facts like the latter fully ground facts like the former. And our take on the grounding profile of (1) suggests another possibility—facts that don't concern individuals that fall outside the subject matter of microphysics are incompletable grounds for existence facts that do concern such individuals.

⁵ See Rubenstein (2023) for a defense of this view.

I'm not alone in thinking that facts involving totality are relevant to incompletable grounding. Let Δ be all the first-order facts that obtain. Rather than claiming that (3) is an incompletable ground for (1), you might, following Leuenberger (2020), propose that Δ is such a ground for [$T(\Delta, being \ a \ first-order \ fact$)]. (T, you will recall, is the "totaling" relation.) Plausibly, we should think that the latter fact obtains only if we have a plausible story to tell about how it might determine/explain the quantificational fact that any first-order fact that obtains is one of thus-and-such facts. But, given our discussion of (1) and (5) above, it's unclear how this story might go.

For another proposal, Trogdon and Witmer (2021) suggest that, while (6) partially grounds (1), the latter has no full ground. But, if (6) partially grounds (1), this is presumably because the (6)-facts fully ground the (3)-facts, and (3) in turn partially grounds (1). So, it's natural to think that, if (6) is an incompletable ground for (1) as Trogdon and Witmer maintain, this is because (3) itself is such a ground for (1). Better, then, to focus on (3)—it's a clearer and more central case of an incompletable ground.

3. Ontological economy

The overall case for incompletable grounding is reasonably strong. Now I trace out some interesting implications. Consider the prima facie plausible view that the ontological economy of a theory is measured not by the entities (including individuals and properties) it posits but the *fundamental* entities it posits in particular (Schaffer 2015). A standard view is that an entity is fundamental if some fundamental fact concerns that entity (deRosset 2013; Rosen 2010). So, if the fact that *a* is *F* is fundamental, this view says that *a* and *F*, entities this fundamental fact concerns, are themselves fundamental.

Here is how this approach to ontological economy is supposed to work. Suppose that a theory is committed to an entity *e*, and the theory has it that *e* is non-fundamental. So, by the lights of this theory, any fact concerning *e* that obtains is partially grounded. According to (*metaphysical*) foundationalism, any partially grounded fact is fully grounded by fundamental facts (Dixon 2016). Given foundationalism, a consequence of the theory is that any fact concerning *e* that obtains is fully grounded by fundamental facts. Hence, the theory is also committed to whatever entities these fundamental facts concern, if any. Importantly, none of these facts concern *e*; otherwise, *e* would be fundamental. By the measure of ontological economy, these further commitments have a cost for the theory but *e* adds nothing to the bill.

This approach to ontological economy presupposes foundationalism. But, as Leuenberger (2020) and Trogdon and Witmer (2022) point out, if there are facts with partial but no full grounds, then foundationalism is false. I've argued that, while (3) partially (1), the former isn't part of a full ground for the latter. So, provided that (1) lacks full grounds altogether (as seems to be the case), the fundamentality approach to ontological economy as formulated doesn't work.

There are at least two ways we might revise the fundamentality approach given the falsity of foundationalism. First, we might introduce a *relativized* notion of ontological cost. Let's say that a fact is *strongly derivative* just in case it's fully grounded, and *weakly derivative* just in case it has partial but no full grounds. Consider again (1) and an individual this fact concerns, a_1 . And consider a theory T_1 according to which a_1 exists and (1) is weakly derivative. In this case, a_1 has a cost for T_1 *relative to totality*, as the theory says that no collection of fundamental facts fully grounds (1). However, this theory might have a different take on the grounding profile of other facts concerning a_1 . Suppose that T_1 also says that the fact that a_1 exists is strongly derivative (i.e., it's fully grounded) and lacks weakly derivative grounds (i.e., no ground for the fact is itself partially but not fully grounded). Let *foundationalism*₁ be the thesis that any strongly derivative fact without weakly derivative grounds is fully grounded by fundamental facts.⁶ Given foundationalism₁, a consequence of T_1 is that the fact that a_1 exists is fully grounded by fundamental facts. In this case, a_1 doesn't have a cost for the theory *relative to existence*.

Second, we might develop a *degreed* notion of ontological cost. Here's the idea. We don't want to say that a_1 has *no cost* for T_1 by the measure of ontological economy. After all, T_1 says that (1)—a fact that concerns a_1 —obtains and isn't fully grounded by fundamental facts. Suppose that T_1 also says that any fact concerning a_1 that obtains is partially grounded. Let *foundationalism*₂ be the thesis that any partially grounded fact is partially grounded by fundamental facts. Given foundationalism₂, a consequence of T_1 is that any fact concerning a_1 that obtains is partially grounded by fundamental facts. In this case, a_1 has a *discounted cost* for T_1 , as all aspects of this entity are partly but not fully rooted in fundamental, then a_1 has a *full cost* for T_1 .

I suggest that we combine both ideas and work with a notion of ontological cost that is relativized and degreed along the lines suggested above. As for totality, the idea is that a_1 has a *discounted totality cost* for T_1 given foundationalism₂. As for existence, if T_1 says that the fact that a_1 exists is fully grounded and lacks weakly derivative grounds, then a_1 has *no existence cost* for the theory given foundationalism₁. And if T_1 instead says that the fact that a_1 exists is full exists is full existence cost for the theory.

In closing, I'll consider another case that further illustrates the approach. Suppose that a theory T_2 is committed to various experiential properties, the *E*-properties, such as the felt dimension of being in pain. Let's focus on *instantiation* facts about the *E*-properties in particular (e.g., the fact that thus-and-such individual has a painful experience). And suppose that T_2 says that each *E*-property is instantiated, and, for any instantiation fact concerning any *E*-property, if that fact obtains, then it's strongly derivative and lacks grounds that themselves are weakly derivative. Given foundationalism₁, a consequence of T_2 is that for, for any *E*-property, some instantiation

⁶ Why the qualification about lacking weakly derivative grounds? For any fact distinct from (1), if that fact obtains, then so too does the conjunctive fact that has (1) and this fact as conjuncts. This conjunctive fact is fully grounded by its conjuncts. Yet the conjunctive fact isn't fully grounded by fundamental facts, as (1) is weakly derivative.

fact concerning that property obtains, and any instantiation fact concerning any *E*-property that obtains is fully grounded by fundamental facts. T_2 is therefore committed not only to the *E*-properties but also to whatever properties these fundamental facts concern. Suppose that T_2 has it that each of these further properties, each *N*-property, is instantiated and non-mental in nature. And, for each *N*, some instantiation fact concerning that property is fundamental. In this case, each *N*-property has a full instantiation cost for T_2 , while each *E*-property has no instantiation cost for the theory.

Given foundationalism₁ and a grounding approach to physicalism (about the mental), I take it that physicalism is true only if something like T_2 so interpreted is true. But suppose that, given the familiar challenges to physicalism, you think that some instantiation facts about the relevant experiential properties that obtain aren't fully grounded. Provided that each N is such that some instantiation fact about that property is fundamental, you might think that the only alternative to T_2 is *fundamentalism* (about the mental), the theory according to which the fundamental facts include instantiation facts about mental properties. For fundamentalism, there are mental properties in addition to non-mental properties that have a full instantiation cost. This theory is going to be too expensive for many.

However, there is another alternative to T_2 . Suppose that a competing theory T_3 is also committed to the *E*-properties. T_3 says that each *E*-property is instantiated. And suppose that T_3 also says that, for any instantiation fact concerning any *E*-property, if that fact obtains, then it's weakly derivative. Given foundationalism₂, a consequence of T_3 is that, for any *E*-property, some instantiation fact concerning that property obtains, and any instantiation fact concerning any *E*-property that obtains is partially grounded by fundamental facts.

 T_3 is also committed to the properties these fundamental facts concern. Suppose that, like T_2 , T_3 has it that each of these further properties, each *N*-property, is instantiated and non-mental in nature. And, for each *N*, some instantiation fact concerning that property is fundamental. Like T_2 , each *N*-property has an instantiation cost for T_3 . Unlike T_2 , each *E*-property does as well. But plausibly the instantiation cost of any given *E*-property is *lower* than the instantiation cost of any given *N*-property for T_3 . This is because for T_3 there is a cost-relevant asymmetry between the grounding profile of obtaining instantiation facts concerning the *E*-properties and *N*-properties. According to T_3 , for each *E*-property, any instantiation fact concerning that property that obtains is partially grounded by fundamental facts. But, for each *N*-property, there is some instantiation fact concerning that property has a discounted instantiation cost, while each *E*-property has a discounted instantiation cost. Overall, T_3 is less expensive than fundamentalism, all other things being equal.

Following Leuenberger (2015), Wilson (2018) considers the idea that emergence is to be understood in terms of incompletable grounding. Given our discussion above, one way to characterize emergence is like this: *P*-properties emerge from *Q*-properties if some instantiation facts concerning *P*-properties lack full grounds and are partially grounded by facts concerning *Q*-properties. Given this take on emergence, the alternative to fundamentalism above in effect says that the experiential properties emerge from the relevant non-mental properties. Let *emergentism* (about the mental) be the dualist theory according to which some mental properties are emergent with respect to non-mental properties in this sense. An upshot of our discussion is that not all naturalistic versions of dualism (with respect to the mental) are on a par with respect to ontological economy, as emergentism has a lower ontological cost than fundamentalism, all other things being equal.⁷

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